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Particle Production by Charges in Condensed Matter

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The rate of electron–positron pair creation and rho–meson creation by uniform quantum motion of charges in condensed matter is calculated in the framework of the Schwinger source theory.

The creation of particle–antiparticle pairs when sufficient energy and momentum is available is a well-known effect in the theory of interacting quantum fields. Quantum electrodynamics enables us to calculate the rate at which the pairs are produced by electrodynamic interactions. In this paper we are interested in pair creation and rho–meson creation by uniform quantum mechanical motion of charges in condensed matter. This problem represents the quantum-mechanical extension of the simpler problem concerning particle production by the classical uniform motion of charges in dielectric medium (Pardy, 1983).

There are many approaches to the formulation of the quantum mechanical theory of pair creation. We use here the Schwinger source theory (Schwinger, 1970, 1973) together with the mass operator method (Schwinger, 1973; Tsai and Yildiz, 1973; Tsai, 1973, 1974) which was also applied to the Čerenkov radiation problem (Schwinger et al., 1976), the analog of our problem of particle creation by moving charges in condensed matter. The Schwinger mass operator method bypasses the explicit use of a wave function and the summation over final states. We will use this method for computing the rate of pair creation and rho–meson production by charges moving uniformly in condensed matter.

There exist two kinds of production of pairs, namely, direct and indirect production. The first process involves only the production of pairs and the second one involves two steps: first the production of γ radiation

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and then its materialization in the external field. Our problem will consist in direct pair production.

The basic quantity which describes the radiation process of photons is the mass operator

$$M(x', x'') = ie^2\gamma^\mu G_+(x', x'')D_{+\mu\nu}(x' - x'')\gamma^\nu + \text{C.T.} \tag{1}$$

where G_+ and $D_{+\mu\nu}$ are the electron and photon propagation functions and as we will suppose the charged particle moves at constant velocity we can write for G_+

$$G_+(p) = (m + \gamma p)^{-1} \tag{2}$$

The symbol C.T. represents the contact term which will be determined later. For the photon propagation function $D_+^{\mu\nu}$ we will use here the following representation (Schwinger et al., 1976):

$$D_+^{\mu\nu}(x - x') = \frac{\mu}{c}(g^{\mu\nu} + (1 - n^{-2})\eta^\mu\eta^\nu)D_+(x - x') \tag{3}$$

where

$$D_+(x - x') = \int \frac{(dk)}{(2\pi)^4} e^{ik(x - x')}D(k) \tag{4}$$

with

$$D(k) = \frac{1}{|\mathbf{k}|^2 - n^2(k^0)^2 - i\varepsilon} \tag{5}$$

and with $\eta^\mu \equiv (1, \mathbf{0})$. Further, μ is the magnetic permeability and n is the index of refraction and $n = (\mu\varepsilon)^{1/2}$, where ε is the dielectric permittivity of condensed matter.

The total decay rate may be inferred from the imaginary part of M via

$$\Gamma = -\frac{2m}{E}\operatorname{Im} M \tag{6}$$

The term $\operatorname{Im} M$ can be decomposed into terms for various different processes including radiation of real photons, direct production of e^+e^- pairs, meson production, etc., which can be achieved by adding to $D(k)$ the term with radiation corrections via

$$\begin{aligned} D(k) &\to D(k) + \delta D(k) \\ &= \frac{1}{|\mathbf{k}|^2 - n^2(k^0)^2 - i\varepsilon} + \int dM^2 \frac{a(M^2)}{|\mathbf{k}|^2 - n^2(k^0)^2 + M^2 - i\varepsilon} \end{aligned} \tag{7}$$

where the last term in Eq. (7) was derived on the condition that $|\mathbf{k}|^2 - n^2(k^0)^2 = -M^2$. The function $a(M^2)$ can be specified as

$$a(M^2) = \frac{\alpha}{3\pi} \cdot \frac{1}{M^2}\left(1 + \frac{2m^2}{M^2}\right)\left(1 - \frac{4m^2}{M^2}\right)^{1/2} \tag{8}$$

(Dittrich, 1978) for the direct production of e^+e^- pairs by electromagnetic interactions.

After insertion of Eq. (7) into Eq. (6) we have for the total rate of the particle production

$$\Gamma = \Gamma_\gamma + \Gamma_{e^+e^-} \tag{9}$$

where Γ_γ is the rate of γ production and $\Gamma_{e^+e^-}$ is rate of e^+e^- production.

Now, we can approach the calculation of the rate of electron–positron pair production. We can write in the momentum representation that

$$M_{e^+e^-} = ie^2\gamma^\mu \langle G_+(p-k)\,\delta D_{+\mu\nu}(k)\rangle\,\gamma^\nu + \text{C.T.} \tag{10}$$

where

$$\langle f(k)\rangle = \int \frac{(dk)}{(2\pi)^4} f(k) \tag{11}$$

After substitution of Eq. (2) and Eq. (3) into Eq. (10) we have with regard to Eq. (7)

$$\begin{aligned} M_{e^+e^-} = {} & ie^2 \int_{4m^2}^{\infty} dM^2 a(M^2) \cdot \frac{\mu}{n^2} \\ & \times \left\langle \frac{1}{k^2 - 2kp} \cdot \frac{1}{n^{-2}|\mathbf{k}|^2 - (k^0)^2 + M^2 n^{-2}} F \right\rangle + \text{C.T.} \end{aligned} \tag{12}$$

where

$$F = \gamma^\mu(m - \gamma(p-k))\left(g_{\mu\nu} + (1 - n^{-2})\eta_\mu\eta_\nu\right)\gamma^\nu \tag{13}$$

Using the representation (Schwinger et al., 1976)

$$\frac{1}{k^2 - 2kp} \cdot \frac{1}{n^{-2}|\mathbf{k}|^2 - (k^0)^2 + n^{-2}M^2} = -\int_0^\infty s\,ds \int_0^1 dt\, e^{-is\chi(t)} e^{-is(1-t)M^2 n^{-2}} \tag{14}$$

where

$$\chi(t) = \chi_0 + \chi_1 \tag{15}$$

$$\chi_0 = \delta^{-1}(tE)^2(1-n^{-2})(t-t_0) \tag{16}$$

$$\chi_1 = \delta(\mathbf{k} - t\delta^{-1}\mathbf{p})^2 - \left[k^0 - (tE)^2\right] \tag{17}$$

$$\delta = t + n^{-2}(1-t) \tag{18}$$

$$t_0 = (n^2-1)^{-1}(n^2\beta^2 - 1) \tag{19}$$

we get

$$\langle e^{-is\chi_1} \rangle = -\frac{i}{(4\pi)^2 s^2 \delta^{3/2}} \tag{20}$$

and

$$\langle e^{-is\chi_1} F \rangle = \langle e^{-is\chi_1} \rangle \cdot \frac{2m\beta^2}{1-\beta^2} \times \left\{1 - (n\beta)^{-2} + 2^{-1}t(n\beta)^{-2}\left[\delta^{-1}\beta^2(1+n^2) + 1 - 3n^2\right]\right\} \tag{21}$$

and for $M_{e^+e^-}$ we have

$$M_{e^+e^-} = -\frac{\alpha}{2\pi} \cdot \frac{\mu}{n^2} \frac{m\beta^2}{1-\beta^2} \int_{4m^2}^{\infty} dM^2 a(M^2) \int_0^{\infty} \frac{ds}{s} \int_0^1 dt\, \delta^{-3/2} \times \left(\exp(-is\chi_0) \cdot \exp\left[-is(1-t)n^{-2}M^2\right] + \text{C.T.}\right) \times \left\{1 - (n\beta)^{-2} + 2^{-1}t(n\beta)^{-2}\left[\delta^{-1}\beta^2(1+n^2) + 1 - 3n^2\right]\right\} \tag{22}$$

At this stage we can determine the contact term. It is determined by the physical requirement that for $n = 1$ the charge radiates no electron–positron pairs. With regard to this condition we have

$$\text{C.T.} = -e^{-is\kappa(t)} \tag{23}$$

where

$$\kappa(t) = (tE)^2(1-\beta^2) + (1-t)M^2 \tag{24}$$

Using the well-known relation (Bjorken and Drell, 1965)

$$\int_0^\infty \frac{ds}{s}\left(e^{isA}-e^{isB}\right)=\ln\frac{B}{A}=\ln\left|\frac{B}{A}\right|+i\arg\frac{B}{A} \tag{25}$$

we have after s integration

$$\int_0^\infty \frac{ds}{s}\left\{\exp\left[-is\left(\chi_0+(1-t)M^2n^{-2}\right]-\exp\left[-is\kappa(t)\right]\right\}\right.$$

$$=\ln\frac{n^2\kappa(t)}{n^2\chi_0+(1-t)M^2} \tag{26}$$

We are interested only in the nonzero imaginary part of Eq. (26). Therefore for $0<t<1$, we must have

$$\kappa(t)>0,\qquad n^2\chi_0(t)+(1-t)M^2<0 \tag{27}$$

and the imaginary part of Eq. (26) is equal to π. From the second inequality of (27) follows the Čerenkov threshold condition $n\beta>1$.

After t integration of Eq. (22) we have for $\Gamma_{e^+e^-}$

$$\Gamma_{e^+e^-}=\frac{m^2}{E}\alpha\frac{\mu}{n^2}\frac{\beta^2}{(1-\beta^2)}\int_{4m^2}^\infty dM^2a(M^2)$$

$$\times\left\{\delta^{-1/2}\frac{2n^2}{n^2-1}\left[a(1-n^2)-bn^2+c\right]+\delta^{1/2}\frac{2n^4}{(n^2-1)^2}c\right.$$

$$\left.+\ \delta^{-3/2}\frac{2n^2}{3(n^2-1)}b\right\}\Bigg|_{t(M^2)}^{1} \tag{28}$$

where $t(M^2)$ is the solution of the algebraic equation:

$$(tE)^2\delta^{-1}(n^2-1)(t-t_0)+(1-t)M^2=0 \tag{29}$$

where $0<t(M^2)<1$ and

$$a=\left(1-(n\beta)^{-2}\right),\qquad b=2^{-1}n^{-2}(1+n^2),\qquad c=2^{-1}(n\beta)^{-2}(1-3n^2) \tag{30}$$

In the case of rho–meson production the spectral function $a_\rho(M^2)$ cannot be determined from Q.E.D. because this process involves strong interaction dynamics. It was calculated by Schwinger and is quoted by Dass (1981) in his work as follows:

$$a_\rho(M^2) = \frac{e^2}{g^2}\delta\left(M^2 - m_\rho^2\right) \tag{31}$$

where g is the coupling constant and m_ρ is the mass of rho–meson. Then, the rate of rho–meson production is

$$\Gamma_\rho = \frac{m^2}{E}\cdot\frac{\alpha\mu}{n^2}\frac{\beta^2}{\left(1-\beta^2\right)}\left\{\delta^{-1/2}\frac{2n^2}{n^2-1}\left[a\left(1-n^2\right)-bn^2+c\right]\right.$$
$$\left.+\,\delta^{1/2}\frac{2n^4}{\left(n^2-1\right)^2}c+\delta^{-3/2}\frac{2n^2}{3\left(n^2-1\right)}b\right\}\Bigg|_{t(m_\rho^2)}^{1} \tag{32}$$

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